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In an earlier report[1], the modus operandi for attacking the problem of light propagation in a magnetic field was set forth. With regard to the more or less obvious approach of developing a magnetic Delbrück s-matrix theory similar to the already known electrostatic Delbrück, a recent publication by Kaitna and Urban[2] provides testimonial to the intractable nature of this formalism.

Thus, the new scheme suggested of replacing the overt interaction of light with the vacuum in the presence of an external magnetic field (symbolically referred to as $\gamma$-H) by the equivalent $\gamma$-$\gamma$ stimulated process assumes greater significance. The present disclosure concerns primarily this aspect of dealing with the red-shift of light in transit through a magnetic field.

The researches of Gatland, Gold and Moffat[3] form the background for the stimulated photon-photon scattering in which the vacuum polarization plays a vital role. The magnetic field is replaced by an effective radiation field associated with the cyclotron resonances. The central question then arises as to the density of such photons required to simulate the strength of the prevailing magnetic field.

Conceptually, it appeared that the wave functions for the Dirac equation embodying the magnetic vector potential should reflect the relative number density of photons for the fundamental and harmonics of the cyclotron radiation. In considering the process of pair production in a d.c. magnetic field, Robl[4] elaborated the necessary eigenfunctions; from the normalization factor, the various densities of cyclotron photons can be estimated. The unit of quantized flux enters in an important way in defining the photon densities for the different frequency components of the radiation field. It will be shown that the contribution of each cyclotron resonance to the stimulated cross section falls off faster than $^{1}/n!$ where $n = 0,1,2,3 \ldots$ for the diverse discrete frequencies.

## The Stimulated Photon-Photon Scattering Model for the Interaction of Light with a Magnetic Field

The d.c. magnetic field can be replaced by a photon field of energy $\omega_i$ and density $n_i$ in all directions. Atomic units are employed with $\hbar = c = 1$. Fig. 1 describes the circumstance of a photon beam of energy $\omega_o$ traversing the region permeated by cyclotron frequency photons

$$\omega_i = (n + \frac{1}{2})\,\omega_c \quad , \quad n = o, 1, 2, 3, \ldots \tag{1}$$

which follows from the Landau levels for electrons (or positrons) in a magnetic field. It is assumed that $\omega_o >> \omega$ with the supposition that stimulation occurs when $\omega' \lesssim \omega$ (this may be an overestimate), i.e. $\Delta\omega = \omega$. Since $\gamma$-$\gamma$ scattering is predominantly s-wave an average over $\theta$ is reasonable. The red-shift, $\omega_R$, is then of the order of $\omega_i$.

In a manner already described[3], the formalism for $\gamma$-$\gamma$ interaction based upon the closed loop Feynman graphs can be invoked to allow for the role of the vacuum polarization. Employing the Lorentz variables $S = (P_1 + P_2)^2$, $t = (P_1 - P_3)^2$, the modulus squared amplitude in standard notation[5] is given by

$$| < M > |^2 = \frac{139}{8100}\left(\frac{\alpha}{2\pi^2}\right)^2 \cdot \frac{r_o}{m^2}\left(\frac{S}{4m^2}\right)^2 \left[3 + \left(1 + \frac{2t}{S}\right)^2\right]^2 \tag{2}$$

Noting that $\left|1 + \frac{2t}{S}\right| \lesssim 1$, and since

$$S = 2\omega_o \; \omega_i \; (1-\cos\theta) \tag{3}$$

the part of the $\gamma$-$\gamma$ cross-section which can be stimulated is

$$\sigma = \frac{\sqrt{2}\;\pi^3\;\omega_i^2}{|\; 1-\cos\theta \;|} \; | < M > |^2 \tag{4}$$

the stimulation factor being

$$\rho = \frac{n_i}{\frac{4}{3}\pi\omega_i^3} \tag{5}$$

Hence, the total stimulated cross section becomes

$$\rho\sigma = \left(\frac{139\sqrt{2}}{1200\pi^2}\right)(\alpha^2\; r_o^2)\; \frac{n_i\;\omega_o^2\;\omega_i}{m^6}\; |\; 1-\cos\theta \;| \tag{6}$$

The number of scattering events per unit path for each $\omega_o$ photon in the presence of the $\omega_i$ photons when averaged over all angles of $\theta$ ($|1 - \cos\theta| \sim \frac{1}{\pi}$) is found to be

$$\rho\sigma n_i = \left( \frac{139 \sqrt{2}}{1200 \ \pi^3} \right) (\alpha^2 r_o^2) \left( \frac{\omega_o^2 \ \omega_i}{m^6} \right) \ n_i^2 \tag{7}$$

To allow for all i kinds of photons as specified by (1), both relations (6) and (7) must be summed appropriately. Thus

$$(\rho\sigma)_{total} = \left( \frac{139 \sqrt{2}}{1200 \ \pi^3} \right) (\alpha^2 r_o^2 \ ) \ \frac{\omega_o^2}{m^6} \sum_i n_i \ \omega_i \tag{8}$$

with a similar expression for $(\rho\sigma n_i)_{total}$ so that the mean free path $\lambda$ can be expressed as

$$\lambda^{-1} = \left( \frac{139 \sqrt{2}}{1200 \ \pi^3} \right) (\alpha^2 r_o^2) \ \frac{\omega_o^2}{m^6} \sum_i n_i^2 \ \omega_i \tag{9}$$

Next the energy loss per unit path length follows:

$$\frac{d\omega_o}{dr} = \sum_i \rho\sigma n_i \ \omega_R \ \approx \sum_i \rho\sigma \ \ n_i \ \omega_i \ = \ \frac{139 \sqrt{2}}{1200 \ \pi^3} \ (\alpha^2 r_o{}^2) \ \frac{\omega_o^2}{m^6} \ \sum_i \ (n_i \omega_i)^2 \tag{10}$$

This result implies a red-shift arising from the energy degradation. The method of analytic continuation will permit specification of the index of refraction associated with the absorptive loss.

## Identification of $n_i$ the Number Density of $i^{th}$-Type Photons

To proceed further with the equivalent model outlined for the vacuum polarization produced by light in the presence of a d.c. magnetic field, it is essential to characterize $n_i$. This can be achieved by referring to the wave functions associated with the relativistic Landau levels deduced from the Dirac equation. The configuration where the magnetic field is orthogonal to the direction of light propagation (as shown in Figure 2) gives rise to eigenfunctions whose normalization factors are prescribed by[4]

$$N_n = \frac{i^{n}}{(\pi^{1/2} 2^n n!)^{1/2} \, L\lambda_f^{1/2}} \tag{11}$$

where n corresponds to the integers in (1) for the Landau energy levels and L denotes the beam width. The parameter $\lambda_f$ represents the characteristic length associated with the quantized magnetic flux

$$\lambda_f^2 = \frac{\hbar c}{He} \tag{12}$$

It is required that $H\lambda_f^2 \gtrsim \hbar c/e = \Phi$ with the smallest value of the flux $\Phi$ some $10^{-7}$ oersted-cm$^2$.

Now the square of the wave function amplitude relates to the $\omega_i$ density so that*



* See Appendix

$$n_i \sim | N_n^2 | = \frac{1}{2^n n! \sqrt{\pi} \; L^2 \lambda_f} \tag{13}$$

Consequently the relations (6) - (10) may be explicitly evaluated with the resultant findings:

$$n_i \omega_i = \frac{n + \frac{1}{2}}{2^n n! \sqrt{\pi}} \cdot \frac{\alpha^{1/2}}{L^2} \cdot \frac{e^{1/2}}{m} \cdot H^{3/2} \tag{14}$$

$$n_i^2 \omega_i = \frac{(n + \frac{1}{2})}{2^{2n} (n!)^2 \pi} \cdot \frac{\alpha}{L^4} \cdot \frac{1}{m} \cdot H^2 \tag{15}$$

The final result for the stimulated cross section takes the form

$$\rho\sigma = \frac{139 \sqrt{2}}{1200 \pi^{5/2}} \frac{n + \frac{1}{2}}{2^n n!} \alpha^{5/2} r_o^2 \frac{\omega_o^2}{m^7} \cdot \frac{e^{1/2}}{L^2} \cdot H^{3/2} \, | 1-\cos \theta | \tag{16}$$

revealing a 3/2 power dependence upon the external magnetic field; each cyclotron harmonic contributes an increasingly smaller amount to the integrated stimulated cross section

$$(\rho\sigma)_{total} = \frac{139 \sqrt{2}}{1200 \pi^{7/2}} \cdot \alpha^{5/2} r_o^2 \cdot \frac{\omega_o^2}{m^7} \cdot \frac{e^{1/2}}{L^2} \cdot H^{3/2} \sum_{n=o}^{\infty} \frac{n + \frac{1}{2}}{2^n n!} \tag{17}$$

The individual contributions to the absorption may be written as

$$\rho\sigma\, n_i = F\,.(\omega_o H)^2\, \frac{n+\frac{1}{2}}{2^{2n}(n!)^2} \tag{18}$$

where F in the abbreviation

$$F = \frac{139\,\sqrt{2}}{1200\,\pi^4}\,.\,\alpha^{5/2} r_o^2\, \frac{e^2}{m^7 L^4} \tag{18a}$$

whence the mean free path becomes

$$\lambda^{-1} = F\,.\,(\omega_o H)^2 \sum_{n=o}^{\infty} \frac{n+\frac{1}{2}}{2^{2n}(n!)^2} \tag{19}$$

The incident photon beam **attenuates** with the square of $\omega_o H$ with the major entry of the fundamental and lower harmonics of the cyclotron resonances.

The energy degradation can then be expressed as follows:

$$\frac{d\omega_o}{dr} = F.\,(me)^{-1}\,\omega_o^2\, H^3 \sum_{n=o}^{\infty} \frac{(n+\frac{1}{2})^2}{2^{2n}(n!)^2} \tag{20}$$

which is the required result for estimating the red-shift.

## Concluding Remarks

At this stage it is evidenced that the virtually intractable external field problem of photon scattering can be replaced by a more lucid stimulated photon-photon process. The calculations are well on the way to see ahead to the ultimate appraisal of the red-shift and the experimental requirements for its observation. Nonetheless, the outlines of the role played by the controllable parameters of light frequency and magnetic field strength can be perceived.

In connection with the underlying $\gamma$-$\gamma$ interaction, it may be of interest to realize that the possibility of light scattering by light was first experimentally looked for[6,7] some thirty five years ago! It was estimated that the cross-section for $\gamma$-$\gamma$ must lie below $10^{-20}$ $cm^2$ to account for the failure to observe the effect[6]. Quantum electrodynamic theory surely supports such a conclusion.

## References


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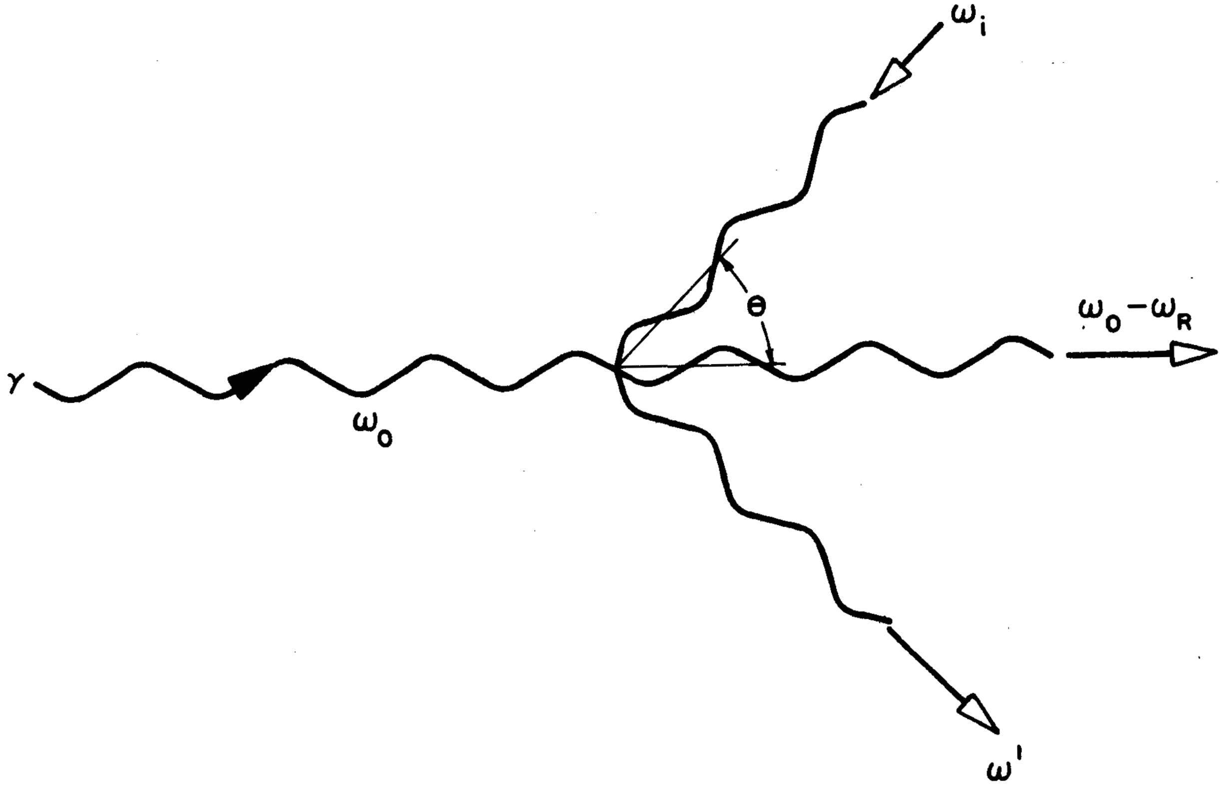


Figure 1

Caption: $\gamma$-$\gamma$ Analogue of the $\gamma$-$\vec{H}$ Interaction in a Vacuum

Magnetic field $\vec{H}$ replaced by an equivalent photon field connected with the cyclotron radiation of the virtual electron-positron pair. $\omega_o$ is the incoming light beam which becomes shifted by an amount $\omega_R$ due to the scattering process indicated.

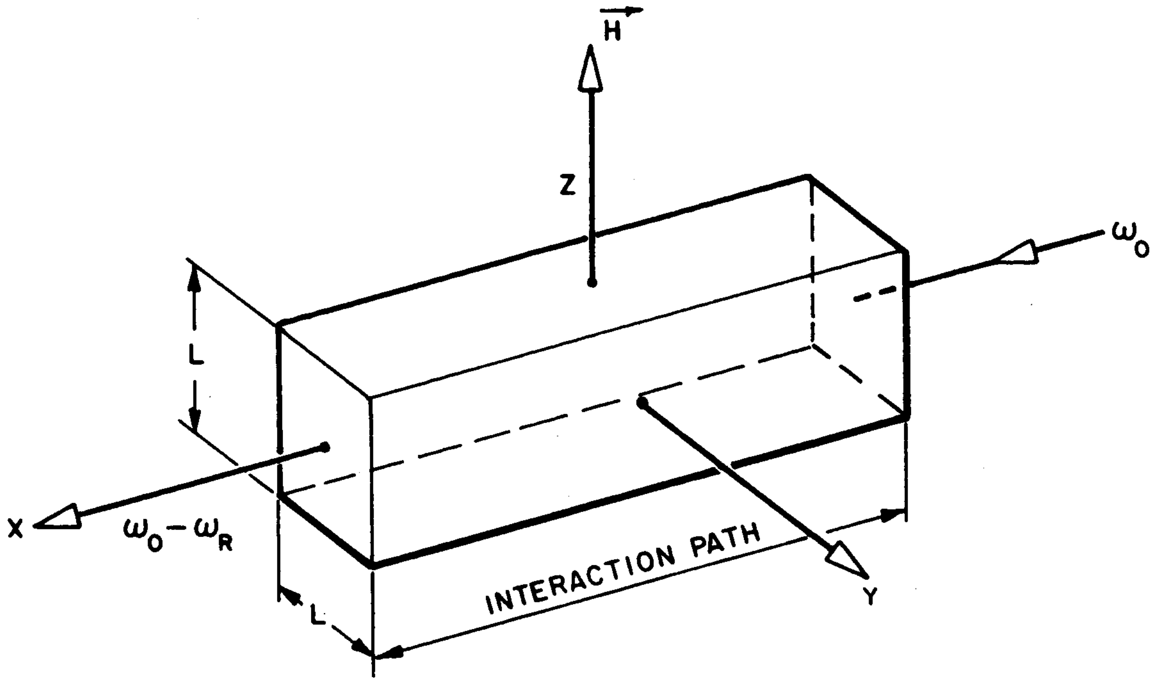


Figure 2

Caption: <u>Orthogonal Configuration of External Magnetic Field $\vec{H}$ with Light Beam $\omega_o$ Having Cross Sectional Area $L^2$.</u>

Dirac equation then contains $A_x = -\frac{1}{2} H_y$, $A_y = \frac{1}{2} H_x$ with $A_z = o$, giving rise to wave functions having normalization factors given by Equation (11).

## Appendix

The scheme offered here allows a first description of $n_i$ to at least determine the form of its behavior. The analysis which follows is to be taken as a first approach to the problem. An improved version based upon a cyclotron radiation field associated with transitions between the various energy levels in (1) will entail matrix elements incorporating the appropriate wave functions for the upper and lower states. But it can be seen that the form of (13) will be preserved. Future study will hopefully clarify and strengthen the theory of the equivalent photon density $n_i$.



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